Optimizing Methods
Seventh List of Problems

1. On the lecture devoted to the introduction of non-linear programming has been formulated the *isoperimetric problem* in a simplified version – for rectangles. Prove directly that the solution of this problem is point $(\frac{L}{4}, \frac{L}{4})$.
2. Formulate the *isoperimetric problem* for the family of the triangles.
3. By using the concept of *convexity* formulated below, prove the *Bruno–Minkovskij inequality*

$$|A+B|^{\frac{1}{2}} \geqslant |A|^{\frac{1}{2}} + |B|^{\frac{1}{2}},$$

where A and B are non-empty subsets of the *Euklidean space* $\mathbf{R}^n$ and $A+B = \{\mathbf{a}+\mathbf{b}\colon \mathbf{a} \in A, \mathbf{b} \in B\}$ for $\mathbf{a} = (a_1, a_2, \ldots, a_n)$, $\mathbf{b} = (b_1, b_2, \ldots, b_n)$.

By definition, we say that a function $f\colon D \to \mathbf{R}$, where $D \subset \mathbf{R}^n$, is *convex* if for every $\mathbf{x}, \mathbf{y} \in D$ and $\lambda \in [0,1]$ the following inequality holds

$$f(\lambda\mathbf{x} + (1-\lambda)\mathbf{y}) < \lambda f(\mathbf{x}) + (1-\lambda) f(\mathbf{y}).$$