Optimizing Methods
Eighth List of Problems

1. Show, by using definition of *convex function* that $f(x) = \ln \frac{1}{x}$, for $x > 0$ is convex.

2. Prove that if f is *convex* then $-f$ is *concave*.

3. Find intervals on which $f(x) = e^{-x^2}$, for $x \in \mathbf{R}$ is convex.

4. Compute all first and second *partial derivatives* for functions

$$f(x, y, z) = xy + ze^x, \;\; g(x, y, z) = \sqrt{x^2 + y^2 + z^2}.$$

5. For $f(x, y, z) = xy + yz + xz$ and $f(x, y, z) = x^2y^2 + y^2z^2 + x^2z^2$ compute $\bigtriangledown f$ and *Hessian* $H(f) = J(\bigtriangledown f)$, where J is *Jacobian* of the vector–valued function.

6. Why the matrix $H(f)$ of task 5 is *symmetric*, i.e. satisfies the condition $H(f) = H^T(f)$?