Fundamentals of Probabilistics Methods Second List of Problems

1. Let $X_i,\ i = 1, 2, \ldots, 10$ be independent random variable, each uniformly distributed over interval $[0, 1]$. By using *Central Limit Theorem* (CLT), calculate an approximation to

$$P\Big(\{\omega \in \Omega\colon \sum_{i=1}^{10} X_i(\omega) > 6\}\Big).$$

2. If 10 fair dice are rolled, find by using CLT, the approximation probability that the sum obtained is between 30 and 40, inclusive.

3. Write CLT for independent random variables $X_1, X_2, \ldots, X_{100}$, where each of them has Poisson distribution with $\lambda = 0,01$.

4. For given join distribution of the random vector (X, Y)

$$\begin{bmatrix} 0,15 & 0,1 & 0,2 \\ 0,1 & 0,1 & 0,1 \\ 0,1 & 0,1 & 0,15 \end{bmatrix},$$

find the distribution of X and Y. Does X and Y are independent?

5. The random variables X and Y are independent and they have the following distributions

X:

-1	0	1
$0,2$	$0,3$	$0,5$

Y:

0	1
$0,3$	$0,7$

Find the distribution of the random vector (X, Y).

6. Find the number $a, b, c \in \mathbf{R}$, that the matrix

$$\begin{bmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ a & b & c \end{bmatrix},$$

represents the join distribution of the random vector (X, Y).

7. Suppose that $X(\Omega) = \{1, 2\}$, $Y(\Omega) = \{-1, 1\}$ and random vector (X, Y) has the join distribution as follows

$$\begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}.$$

Find the distribution of $Z = X + Y$.