# Fundamentals of Probabilistics Methods
# First List of Problems

1. Prove that for every two events $A,\ B$ if $A \subseteq B$ then $P(A) \leqslant P(B)$.

2. By using the formula
$$P(E \cup F) = P(E) + P(F) - P(E \cap F),$$
compute $P(A \cup B \cup C)$, where $A, B, C \in \Sigma$.

3. Prove that $\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = 1$ for every $p \in [0,1]$.

4. We say that two events $A,\ B$ are *stochastically independent* if $P(A \cap B) = P(A)P(B)$. Prove that then $A,\ B^c$, and $A^c,\ B^c$ are also stochastically independent.

5. For random variable $\mathbf{X}$ with *distribution* given by

| $x_i$ | -1 | 0 | 2 | 3 |
|---|---|---|---|---|
| $p_i$ | 0,1 | 0,2 | 0,3 | 0,4 |

find the distribution of the random variable $\mathbf{Y} = 2\mathbf{X} - 1$.

6. $\mathbf{X}$ has distribution as in above problem. For given events
$$A = \{\omega \in \Omega\colon\ -0,5 \leqslant \mathbf{X}(\omega) < 2,5\},$$
$$A = \{\omega \in \Omega\colon\ -1 < \mathbf{X}(\omega) < 2\},$$
$$A = \{\omega \in \Omega\colon\ 0 \leqslant \mathbf{X}(\omega) \leqslant 3\},$$
compute $P(A)$ by using "step" function and an array method.

7. Assume that $\mathbf{X} \in U([1,3])$. Prove that the random variable $0,5(\mathbf{X}-1)$ is *uniformly* distributed on the *unit interval*.

8. We know that the random variable $\mathbf{X}$ has *density probability function* given by
$$f(x) = \begin{cases} 2-2x & \text{dla} \quad x \in (0,1); \\ 0 & \text{dla} \quad x \notin (0,1). \end{cases}$$
Find the *cummulative distribution function* and than compute
$$P(\{\omega \in \Omega\colon\ -0,5 < \mathbf{X}(\omega) < 0,75\}).$$

9. Prove that *standarizing* $\mathbf{X} \in \mathcal{N}(m, \sigma^2)$, we get always *standard normal distribution.*

10. For $\mathbf{X} \in \mathcal{N}(m, \sigma^2)$ with $m = -3$, $\sigma = 2$ compute
$$P(\{\omega \in \Omega\colon\ |\mathbf{X}(\omega)| < 1\}).$$

11. For $\mathbf{X} \in U([a, b])$ compute $E\mathbf{X}$, $var(\mathbf{X})$.

12. For $\mathbf{X} \in \mathcal{W}(\lambda)$ compute $E\mathbf{X}$, $var(\mathbf{X})$.

13. $\mathbf{X}$ has distribution
$$f(x) = \begin{cases} 3x^2 & \text{dla} \quad x \in (0,1); \\ 0 & \text{dla} \quad x \notin (0,1). \end{cases}$$
Compute $var(\mathbf{Y})$ if $\mathbf{Y} = \mathbf{X}^2$.